Math 128a – Week 14 Worksheet
GSI: Izak, (4/28/21)

Section 6.4 Problems

Problem 1. *Find all values of α that make the following matrix singular*

$$A = \begin{pmatrix} 1 & -1 & \alpha \\ 2 & 2 & 1 \\ 0 & \alpha & -\frac{3}{2} \end{pmatrix}$$

Problem 2. *Prove that AB is nonsingular if and only if both A and B are nonsingular*

Problem 3. *It turns out that if A depends on a parameter t in a C^1 way (the components of A are C^1 functions of t), then $\frac{d}{dt}\log|\det A(t)| = Tr(A^{-1}\frac{d}{dt}A)$. Verify this for the matrix:*

$$A(t) = \begin{pmatrix} a(t) & b(t) \\ c(t) & d(t) \end{pmatrix}$$

Section 6.5 Problems

Problem 4. *Solve the linear system of equations:*

$$\begin{pmatrix} 1 & 0 & 0 \\ -2 & 1 & 0 \\ 3 & 0 & 1 \end{pmatrix} \begin{pmatrix} 2 & 1 & -1 \\ 0 & 4 & 2 \\ 0 & 0 & 5 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \\ -5 \end{pmatrix}$$

Problem 5. *Consider the matrix:*

$$A = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 4 & 0 \\ 0 & 1 & -1 \end{pmatrix}$$

Find the permutation matrix P so that PA can be factored into the product LU, where L is lower triangular with 1s on its diagonal and U is upper triangular.

Section 6.6 Problems

Problem 6. *Determine which of the following matrices are symmetric, singular, strictyl diagonally dominant, positive definite:*

1. $\begin{pmatrix} 2 & 1 \\ 1 & 3 \end{pmatrix}$

2. $\begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 0 \\ 1 & 0 & 4 \end{pmatrix}$

3. $\begin{pmatrix} 4 & 2 & 6 \\ 3 & 0 & 7 \\ -2 & -1 & -3 \end{pmatrix}$

4. $\begin{pmatrix} 4 & 0 & 0 & 0 \\ 6 & 7 & 0 & 0 \\ 9 & 11 & 1 & 0 \\ 5 & 4 & 1 & 1 \end{pmatrix}$